10.6 ***Math Focus.*** The EPR Paradox

Quantum mechanics is a paradoxical theory. Many people do not really understand where the paradox lies, however. For example, the popular author Timothy Ferris has written that the mystery of quantum mechanics is the "quantum jumps" by which a particle goes from one place to another without going through the space in between. Quantum mechanics says no such thing. Every particle moves continuously from one place to another. A quantum jump is simply the change that occurs when a particle quickly gains energy.

The randomness in quantum mechanics is not really a paradox, either. We do not know the cause of the randomness, but this is just a statement of our ignorance, not a contradiction.

There *is* an unresolved paradox in quantum mechanics, however. This is called the problem of *nonlocality* and is best presented in terms of the EPR paradox. EPR stands for Einstein, Podalksy, and Rosen, the three scientists who first proposed it.[2] It became even more paradoxical when it was put into its modern form by physicists Bell and Shimony in the 1960's, and the experiment was actually performed in the 1970's and 1980's, so that it was no longer just a hypothetical possibility.

The EPR paradox comes about because of *correlations* of different quantum particles. Some physical processes lead to the emission of two particles which have properties that must be the same. We say these properties are correlated. If we separate the particles over a large distance, they can still have correlated properties. The paradox arises when the distances are so great that it would seem to violate Einstein's theory of relativity if the two particles are still correlated.

Imagine, for example, that you and a friend are in two separate space ships, far from the earth. A third person on earth shoots out to each of you a small package containing a light which can be either red or green. You do not know what color your light will be, but you do know that it must be the same as the one received by your friend. When you get the package, you see that the light is green. You instantly know that your friend's light is green.

This does not seem strange, if you assume that both lights were green when they started out from earth. When you receive the green light, you instantly get knowledge about something thousands of miles away, but no signal really traveled from your friend to you; you already had knowledge that the two lights had to be the same.

What if you were told that the lights were not green during the whole trip, but instead could be either green or red randomly, and did not settle on a final color until you opened the package? Then it would be strange to think that your friend's light must be the same color as yours, as soon as you open the package. The theory of relativity says that nothing can go faster than the speed of light, and therefore, your friend's light cannot get any information about the color of your light until some time has passed after you opened yours. If you are far away that could be weeks or months. If he opens his package and looks at his light

[2]Einstein believed that quantum mechanics was an incomplete theory, and this paradox was one of his arguments for this.

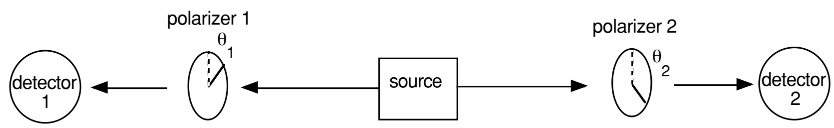

Figure 10.5: Layout of the standard EPR experiment.

before that time has passed, it would seem that the color of his light must be random. If his color is always the same as yours, even if he opens his package without waiting, it would seem that some signal has traveled from one package to the other faster than the speed of light. That would contradict the theory of relativity.

This does not happen with light signals in outer space, but it does seem to happen with certain quantum particles. The experiment in which this is seen is like the one shown in Figure 10.5. A light source consists of atoms which emit two photons at the same time in opposite directions. If the proper type of atom is chosen, then the polarizations of the two photons are always the same. The polarization correlation of the photons can be measured using polarizers and photon detectors at each end, as shown in Figure 10.5. If both photons pass through the polarizers and are detected at the same time by the two different detectors, we call this a *coincidence* of the two detectors.

Because the atoms can turn in any direction, there is no reason why the photons must be polarized in any particular direction—all we know is that the two photons from a single atom must have the *same* polarization.

To start, we set polarizer 1 at $\theta_1 = 0°$ and polarizer 2 at $\theta_2 = 90°$. Suppose that one of the photons hits polarizer 1 first. Since the polarization of the photons is random, we do not know if it will pass through. If it does pass through, however, then we know its polarization afterwards must be the same as the polarizer. We then instantly know that the other photon must have the same polarization, that is, 0°. Therefore, it cannot pass through the other polarizer, because Malus's law says that the probability of passing through the polarizer is proportional to $\cos^2 90°$, which is zero. The same argument holds if a photon passes through polarizer 2 first. In that case, both photons must be polarized at 90°, in which case the other photon can not pass through polarizer 1. We can do this experiment, and find that we get this result: when the polarizers are set at 0 and 90 degrees, we get no coincidences of the photon detectors.

We might imagine that this situation is just like that of two lights being sent from earth already green. Perhaps the polarizations of the photons are not random after all, and all the photons are sent out with polarizations either both along zero degrees, or both along 90 degrees. That would give us the result that we get no coincidences when the polarizers are set at $\theta_1 = 0°$ and $\theta_2 = 90°$.

If this is the case, however, then we can predict what will happen if we change the settings of the two polarizers. Suppose that we set polarizer 1 at $\theta_1 = 45°$ and polarizer 2 at $\theta_2 = -45°$. If each photon is polarized at 0°, then Malus's law says that that probability of each photon passing through its polarizer is $\cos^2 45° = 0.5$, and therefore, the probability of a coincidence is just the product of these probabilities, $(0.5)(0.5) = 0.25$. If both photons are polarized at 90°, we get the same probability of a coincidence, as discussed in Assignment 1. Therefore, if we set the polarizers at $\theta_1 = 45°$ and $\theta_2 = -45°$, and the photons are all polarized along 0° or 90°, we expect to see coincidences of the photon detectors 25% of the time.

If we do the experiment, however, we find that we get *no* coincidences of the detectors in this case. Why not? In this case, the polarizers are still 90° apart. Therefore, the same argument applies as above. The photons are emitted with random polarization, but after one photon passes through the first polarizer, the second one must be the same, and since this is 90° relative to the second polarizer, it cannot pass through.

Of course, we might say that the photons in this second experiment all had polarizations at 45° or −45°, even though they all had polarization of 0° or 90° in the first experiment. After all, we did two different experiments, and did not use the same photons in each case. That would solve the problem of the apparent violation of the theory of relativity, but it means that somehow the atoms must know which way the polarizers are set, and emit photons in a special way depending on how we set the polarizers. That seems even worse, as if nature were secretly working to deceive us. As discussed in Section 1.10, scientists assume that nature is not perversely deceptive.

This type of experiment has been done several times. The photon detectors were not on rocket ships far from earth, but they were far enough apart that no signal could go from one to the other before the photons were counted.[3]

What is the resolution? In logic, when you follow an argument to its logical conclusions, and find a contradiction, then one or more of the premises must be false. This seems to imply that one of our assumptions was false—but which one? We seem to have a choice—one of the following must be true:

- 1. *Quantum mechanics is not entirely correct.* The above argument relies on the assumption that we have calculated the quantum probabilities correctly.
- 2. *Einstein's theory of relativity is not entirely correct.* In particular, faster-than-light communication is possible. As you can imagine, Einstein strongly opposed this idea, which he called "spooky actions at a distance." The experiment described here does not require a signal to be sent from one person to another faster than light, however. Although the particles appear to arrange their behavior instantaneously across large distances, no particle is ever observed to go faster than light. It may be that the rules for relativity apply only to things people can directly observe, and not to the interactions between particles.
- 3. *The experiment was done wrong or interpreted incorrectly.* We cannot rule this out, even though many scientists have examined the experiments carefully. There are always some experimental uncertainties. For example, in the above argument the experimental result of zero coincidences was given for both cases of perpendicular polarizers, but actually there was a large number of accidental coincidences due to

[3]The distances were a few meters, but the timing of the detectors was accurate to less than a billionth of a second. In a billionth of a second, light can only travel 30 cm.

background light, which had to be subtracted. If the number of these accidental coincidences was not calculated correctly, the experiments would give a different answer.

- 4. *It is not proper to make definite statements about what would have happened in experiments you didn't do.* As mentioned above, if the photons were emitted differently in the two different cases of polarizer settings we discussed, then we can not insist that there is a violation of the theory of relativity. It could be that the photons were different in each case. The contradiction only arises if we insist that the *same* set of photons *would* have given us 25% coincidence probability *if* we had set the polarizers at $\theta_1 = 45°$ and $\theta_2 = -45°$.

The problem is that none of these choices make physicists very happy. Quantum mechanics and relativistic mechanics both are highly successful, predictive theories, and no one has found an error in either one yet. At the same time, the experiments discussed above were done by highly competent scientists who are trusted as authorities, and other scientists have analyzed their results without finding any errors. On the other hand, making definite statements about things we haven't seen is part of our daily lives and underlies much of science. We typically think, "If a tree fell in the woods, if I had been there I would have heard a sound." If we say this kind of thinking is wrong, then we should say, "There is no telling whether I would have heard a sound or not." Taking this view makes the world seem spooky and unreal, as though it rigs itself to do things only when we are looking at it. It sounds like the positivism of Mach all over again.

Because of the problems with all of these options, we must simply say that we do not yet have a clear resolution to this paradox. It is one of the great unsolved mysteries of science. Various scientists and philosophers have argued for one or the other of these options, but none of these has been widely accepted. As is often the case, however, people can use a theory without fully understanding all the philosophy of it. The two-particle correlations discussed here are the basis of a new method of passing out secret codes, known as *quantum cryptography.*

Assignment:

1. Verify the calculation above, that if $\theta_1 = 45°$ and $\theta_2 = -45°$, and both photons are polarized at exactly 90°, the probability of a coincidence according to Malus's law is 0.25. Notice that $\cos(-45°) = \cos 45°$ and $\cos 135 = -\cos 45$.

2. Suppose that the two polarizers are not set exactly 90 degrees apart. Show that the probability of a coincidence for any angles θ_1 and θ_2, when at least one photon has passed through a polarizer, is $\cos^2(\theta_1 - \theta_2)$. Start with the same assumption as above, that if one photon has passed through polarizer 1 at angle θ_1, then the other photon must be polarized the same. Then use Malus's law to calculate the probability of that photon passing through polarizer 2.