2.11 *Math Focus.* Math Trick #6: Units in Equations

In the last lesson, we introduced a "funny" unit for force, the Newton, which is given by 1 N = 1 kg-m/s^2. It can be somewhat tricky to work with these types of odd units in equations.

The first thing to keep in mind is that every measurement has two parts: a *number* and a *unit*. You never substitute a number for a unit, or vice versa. Units can be converted to other units, but they are never changed into numbers.

Second, in an equation, if you are substituting a value for a variable, you substitute both *the number and the unit together*. For example, suppose we have the equation

$$x = \frac{1}{2}\left(\frac{F}{m}\right)t^2,$$

and we want to substitute for F, m and t to get x, and we have $F = 6.7$ N, $m = 1.5$ kg, and $t = 8$ s. We write

$$x = \frac{1}{2}\frac{(6.7 \text{ kg-m/s}^2)}{1.5 \text{ kg}}(8 \text{ s})^2.$$

Next, if you are multiplying or dividing two numbers with units, you multiply or divide the *numbers* with each other, and multiply or divide the *units* with each other separately. So in our above example,

$$x = \frac{1}{2}\frac{(6.7)(8)}{1.5}\frac{(\text{kg-m/s}^2)(\text{s})^2}{\text{kg}}.$$

You must always check that the units of your answer come out right. In the above equation, we can cancel the units of kg and seconds, to get

$$x = 17.8 \text{ m}.$$

Adding and subtracting numbers with units works differently. *If you are adding or subtracting two numbers with units, you can only do it if they have the same units.* If they do, you add or subtract the numbers with each other, and leave the unit the same. So for example, suppose we have

$$x = 10 \text{ mi} + 800 \text{ m}.$$

You can't add miles and meters to each other, but since they are both units of length, you can convert one into the other. You can choose either unit to convert. So, for example,

$$x = 10 \text{ mi} \times \frac{1609 \text{ m}}{\text{mi}} + 800 \text{ m} = (16090 + 800) \text{ m} = 16890 \text{ m}.$$

Assignment:

1. Verify the above calculations.

2. Use Newton's second law, $a = F/m$, to find the force necessary to accelerate an object of 10 kg at 30 m/s^2.