1.4 *Math Focus.* Math Trick #1: Uncertainty and Significant Digits

In the previous sections, we said that no one can be perfectly certain of anything. An important consequence of this is that no number that relates to the real world can be given exactly. Any number relating to the real world must always have an associated "uncertainty." We write numbers in the following form:

$$x \pm \Delta x.$$

The number after the "±" is the uncertainty. For example, we could write $x = 13 \pm 0.5$. This means the person who is reporting the value estimates that the observation which gave the number could have allowed a number anywhere from 12.5 to 13.5 even if the measurement was done right.[6]

Saying the number has uncertainty does not mean it was a "bad" measurement, i.e. that the person made a mistake (calling it "error," as people sometimes do, can therefore be misleading). *Every* measurement must have some uncertainty. For example, what is the most accurate measurement you can make with a ruler? If you look at it, you will see that you cannot be more accurate than the smallest spacing between the lines.

If a number is reported without an uncertainty, then it is commonly assumed that the uncertainty is about the same as the *lowest significant digit*. The lowest significant digit is defined according to the following rules:

Rule 1. If the number has no decimal point, then the lowest significant digit is the lowest digit which is not zero. For example, in the number 13,000 the lowest nonzero digit is 3, which is in the thousands place, so the assumed uncertainty is ±1000.

Rule 2. If the number has a decimal point, the lowest significant digit is the last digit. For example, the number 13,000. has an assumed uncertainty of ±1, the number 0.013 has an assumed uncertainty of ±0.001, and the number 0.0130 has an assumed uncertainty of ±0.0001.

Note that these rules mean that it can be incorrect to give a number with a lot of extra digits if the accuracy of the measurement does not justify it. This can be a temptation if you are using a calculator, because it often gives you a display full of digits.

The *relative uncertainty* is the ratio of the uncertainty to the number itself. For example, (10 ± 2) has a relative uncertainty of $2/10 = .2$, in other words, it is only accurate to within 20%.

There are several rules for reporting the uncertainty of a number which is calculated from two or more other numbers which each have some uncertainty. These rules come from the field of *statistics*, which can become very complicated. We will not go into the math of

[6] The "Δx" in the above means the difference. Δ is the Greek letter *d*. Therefore, Δx means the same thing as dx, that is, a difference between two values, except that Δx is usually used for a larger difference, while dx is assumed to be very small.

statistics, which is very important for understanding real experiments, but we will give just two basic rules:

Statistics Rule #1. *If you are adding or subtracting two numbers, the uncertainty in the answer is the sum of the two uncertainties.* Uncertainties never cancel each other. Even when you are subtracting, the uncertainties add.

For example, $(12 \pm 2) + (8 \pm 1) \simeq (20 \pm 3)$, and $(12 \pm 2) - (8 \pm 1) \simeq (4 \pm 3)$. As you can see, subtracting two numbers can often lead to a large relative uncertainty in the answer.

Statistics Rule #2. *If you are multiplying or dividing two numbers, the relative uncertainty in the answer is the sum of the two relative uncertainties.*

For example, $145 \times 257 = 37300$, *not* 37265. Why? Because the assumed relative uncertainty of the two numbers is about 1/100. Therefore, the answer can only have the same relative uncertainty. If you get extra digits in your answer you must round them off, or else you are being dishonest. Of course, 145.00 times 257.00 equals 37265!

Another way of putting this is when multiplying or dividing, you must keep on the *lowest number of significant digits* of the two numbers. The number of significant digits is the number of digits between the most significant digit and the least significant digit. The most significant digit is defined as the *largest nonzero digit.* So, for example, 13,000 has two significant digits, 0.00133 has three significant digits, and 1.00013 has 7 significant digits.

All this approximation and uncertainty may bother you, but this is an important lesson about science. *We can never be perfectly certain of anything.* We can only estimate when some things are more certain than others. The scientific method gives us ways to reduce uncertainty as much as possible, to be very accurate, although we can never be perfectly accurate.

Assignment:

1. What are the uncertainty and the relative uncertainty in the following numbers?

12000	37 ± 4
12000.	12000 ± 1
12000.001	.00012 ± .00005
.000120012	

2. Do the following operations and give the answer to the proper number of significant digits.

37512 ×456	(400 ± 40) + (300 ± 50)
.00456 × .03	(400 ± 40) - (300 ± 50)
60 / .00456	(400 ± 40) × (300 ± 50)
	(400 ± 40)/(300 ± 50)